Technical Notes

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Combining Global and Local Approximations

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Introduction

THE design optimization of an engineering system typically requires hundreds of analyses of that system. The use of approximations to the objective function and constraints during portions of the design process is quite common, because for very few design problems can we afford the computational cost of hundreds of analyses. Such design approximations can be divided into two classes. First we have local, derivative-based approximations such as the linear approximation based on a Taylor-series expansion about a design point. These approximations are typically based on an accurate model of the system response and its derivatives. Second, we have global approximations that try to capture the behavior of the objective function or constraints over the entire design domain. Such approximations are often based on a simplified theory, a coarser model, or both. Here, such global approximations are referred to as simple-model approximations. Local approximations are typically very accurate near the design point where they are generated, but their accuracy can deteriorate catastrophically at a distance. Simple-model approximations may not be very accurate anywhere in the design space, but, on the other hand, they typically do not experience the catastrophic deterioration in accuracy associated with local approximations.

Consider, for example, approximating the function $f=\sin(\pi x)$ in the interval (0,0.5). A first-order local approximation based on data at the origin is $f_L=\pi x$. A global approximation could be a parabola. Matching the maximum point, we get a global approximation as $f_G=4x(1-x)$. At $x=0.1$ we have $f=0.309$, $f_L=0.314$, and $f_G=0.36$. Clearly, the local approximation is superior. However, if we check far from the origin, at $x=0.4$, $f=0.951$, $f_G=0.96$, and $f_L=1.256$. That is, the local approximation can become very inaccurate if we venture too far from the point where it was constructed.

In the past few years there has been much research into improving local approximations so as to extend their region of usefulness. In structural optimization the idea of intervening variables has been popular. For example, for many structural design problems it was found that the reciprocal approximation, which is a linear approximation in the reciprocal of the design variables, is more accurate than the ordinary linear approximation.[1] Similarly, it was found that forces approximate better than stresses,[2] so that a linear approximation of element forces followed by exact calculation of stresses is more accurate than a linear approximation of the stresses. Similarly, in aerodynamics there have been efforts to improve local approximations for aerodynamic drag, such as coordinate stretching.[3]

There has also been some research into improving global or simple-model approximations. One active area is the use of the so-called reduced-basis approximations in structural dynamics. There, the structural model is reduced to a low-order model by retaining only a small number of vibration modes, and assuming that the structural response can be approximated as a linear combination of these modes. Research has focused on the best selection of these modes[4] as well as on methods that improve accuracy for a given set of modes.[5] It may be expected that the advantages of local approximations and simple-model approximations can be combined. In fact, multigrid analysis methods[6] have demonstrated the utility of working simultaneously with coarse and refined approximations. The objective of this Note is to describe a method for combining local and global approximations, a method herein called the global-local approximation (GLA) method.

Global-Local Approximation

The GLA approach is based on a common method for using inexact global approximations such as those obtained from a coarse discretization of the problem. Specifically, the crude approximation is compared at one design point to a more refined approximation, or to the exact result, if available. The ratio of the refined approximation to the crude one is a scaling factor that is used to multiply the crude approximation at other design points. The scaling factor introduces some local flavor into the global approximation in that it is most effective near the design point where the factor is calculated. For example, consider the sine function just discussed, and assume that we calculate a scaling factor at $x=0.1$. At this point $\sin(\pi x)=0.309$ while the parabolic approximation is equal to 0.36. We can use the ratio of these two numbers as a scaling factor to multiply the approximation and obtain a new approximation of $f_G=3.43x(1-x)$. This approximation will be much more accurate near $x=0.1$ than the original approximation, but will be less accurate at points farther away. For example, at $x=0.5$ the original approximation was exact, while the new approximation has about 14% error.

The GLA approach refines the traditional scaling factor by using a linearly varying scaling factor instead of a constant one. For the sake of simplicity consider first a function of a single variable $f(x)$, which is approximated globally by $f_G(x)$, and define the scaling factor f_f calculated at a design point x_0 as

$$f_f(x_0)=f(x_0)/f_G(x_0) \tag{1}$$

The scaling factor at any other point will be approximated as

$$f_f(x)=f_f(x_0)+(x-x_0)f_f' \tag{2}$$

where prime denotes derivative with respect to x. Using the definition of f_f this can also be written as

$$f_f(x)=\frac{f(x_0)}{f_G(x_0)}\left[1+(x-x_0)\left(\frac{f'(x_0)}{f(x_0)}-\frac{f_G'(x_0)}{f_G(x_0)}\right)\right] \tag{3}$$

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and then the GLA, denoted f_{GL}, is given as

$$f_{GL}(x) = f_f(x) f_G(x) \tag{4}$$

This approach is applicable to any number of variables with Eq. (2) replaced by the first-order Taylor series expansion of f_f.

As an example consider again $f = \sin(\pi x)$ and $f_G = 4x(1-x)$ and $(x_0) = 0.1$. We obtain

$$f_f = 0.858 + 0.780(x - 0.1), \qquad f_{GL} = 4x(1-x) f_f \tag{5}$$

while a linear approximation based on data at $x = 0.1$ gives

$$f_L(x) = 0.309 + 2.988(x - 0.1) \tag{6}$$

Comparing the approximations we note that at $x = 0.2$, close to x_0, we get $f_G = 0.64$, $f_L = 0.608$, $f_{GL} = 0.599$, and $f = 0.587$. Away from x_0, at $x = 0.4$, we get $f_G = 0.96$, $f_L = 1.205$, $f_{GL} = 1.048$, and $f = 0.951$. So that the local-global approximation is better than the local approximation both close to the nominal design point and also far away from it.

Beam Example

The GLA approach can be applied with a variety of global approximations. One of the more common applications is when the approximation involves a coarse discretization of the mathematical model. This is demonstrated for a simple beam problem shown in Fig. 1. The clamped-clamped beam has a step change in cross-sectional area, and the moment of inertia is assumed to vary with the third power of the area. A coarse approximation is provided by using one finite element to represent each half of the beam, while a two-element model of each half is assumed to be accurate enough to be called exact for the purpose of the discussion here. The buckling load and the two lowest vibration frequencies are to be approximated.

The initial design point for the purpose of constructing the approximation has an area ratio $A_1/A_2 = 1.5$ and the approximations are tested by increasing A_1 until $A_1/A_2 = 3$. Figure 2 compares the exact buckling load b, the linear approximation b_L, the global approximation (single element per half beam) b_G, and the GLA b_{GL}. Figures 3 and 4 provide similar comparisons for the first two frequencies of the beam. It is seen that the GLA combines the advantages of the global and local approximation in being very accurate near the nominal point without excessive errors far from that point.

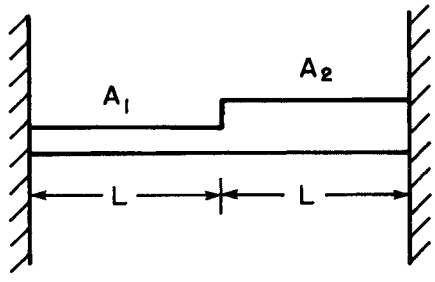

Fig. 1 Step-beam geometry.

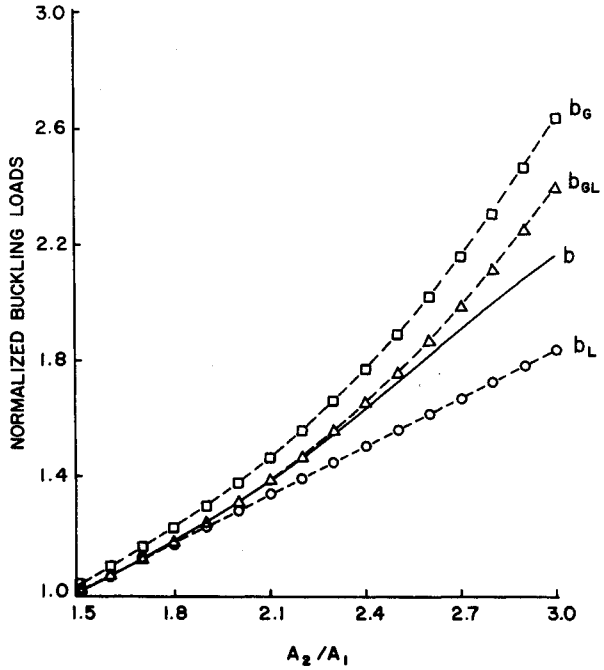

Fig. 2 Approximations to buckling load for step beam.

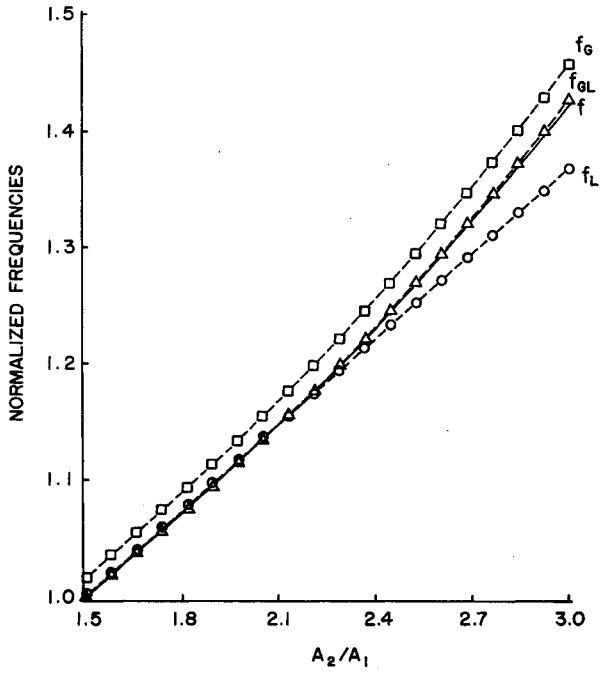

Fig. 3 Approximations to first vibration frequency for step beam.

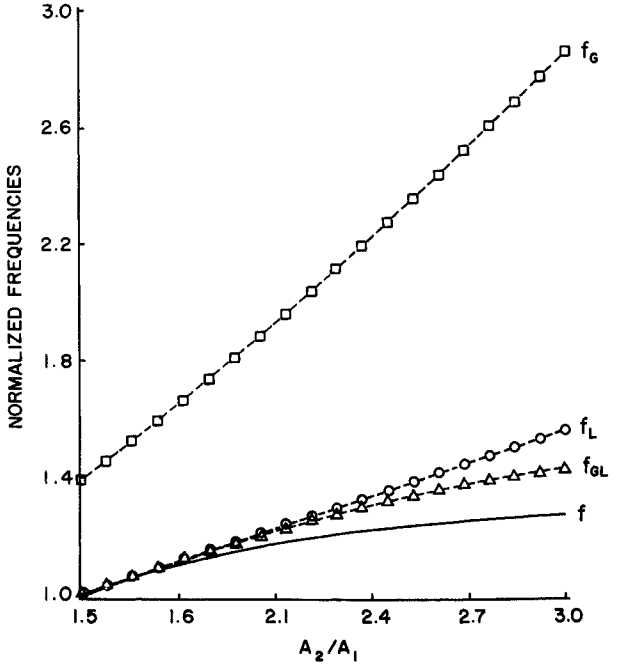

Fig. 4 Approximations to second vibration frequency for step beam.

Concluding Remarks

A global-local approximation based on a linear approximation to a scaling factor has been presented. The approximation permits us to use a global approximation based on a simple model of our problem to extend the range of usefulness of derivative-based approximations to a more refined model. The method was demonstrated for a simple beam example with a crude and more refined finite element model.

Acknowledgment

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Algorithm for Modification of Parameters in Vibrating Systems

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Nomenclature

$\mathfrak{D}_B A$ = derivative of a $p \times q$ matrix function A with respect to an $s \times t$ matrix argument B (see Ref. 2)

$$\frac{\partial A}{\partial B} = \left[\frac{\partial A}{\partial b_{kl}}\right] \quad (k = 1,\ldots,s;\ l = 1,\ldots,t)$$

where

$$\frac{\partial A}{\partial b_{kl}} = \left[\frac{\partial a_{ij}}{\partial b_{kl}}\right] \quad (i = 1,\ldots,p;\ j = 1,\ldots,q)$$

$A \otimes B$ = Kronecker product of a $p \times q$ matrix A and an $s \times t$ matrix B (see Ref. 3)

$$A \otimes B = \begin{bmatrix} a_{11}B & a_{12}B & \ldots & a_{1q}B \\ a_{21}B & a_{22}B & \ldots & a_{2q}B \\ \vdots & \vdots & \ldots & \vdots \\ a_{p1}B & a_{p2}B & \ldots & a_{pq}B \end{bmatrix}$$

I = unit matrix

E^{ij} = ijth elementary matrix; all elements zero except for 1 at the ijth position

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Introduction

THE importance of obtaining sensitivities for an eigenvalue problem stems from the fact that partial derivatives with respect to system parameters are extremely important for effecting efficient design modifications for given situations, for gaining insight into the reasons for discrepancies between structural analyses and dynamic tests, and for indicating system model changes that will improve correlations between analyses and tests. Knowledge of sensitivities can be very useful in the modification of system parameters; however, the changes of these parameters that realize a system possessing prescribed eigenvalues and eigenvectors are still unknown.

This Note presents and demonstrates an efficient method of determining changes in system parameters for predetermined changes of eigenvalues and eigenvectors.

Eigenvalue Problem of Undamped Systems

The eigenvalue problem of a vibrating system described by the equation

$$M\ddot{y} + Ky = 0 \tag{1}$$

where the matrices M and K are symmetric and positive definite and can be written in the matrix form

$$(K - \omega^2 M)\, y = 0 \tag{2}$$

Let us suppose that the system (1) has n distinct eigenvalues. Then we have n linearly independent eigenvectors y_i corresponding to the eigenvalues λ_i. The eigenvalues form the spectral matrix of order n, Λ, and the eigenvectors can be arranged in an $n \times n$ square matrix Y, called the modal matrix. These matrices satisfy the orthogonality given by

$$Y^T M\, Y = I \tag{3}$$

$$Y^T K\, Y = \Lambda \tag{4}$$

Modifications of System Parameters

The modification problem consists of determining changes to the parameters of system (1) represented by the matrices M and K, which realize a new system possessing predetermined eigenvalues and eigenvectors represented by Λ and Y. In other words, we want to determine ΔM and ΔK so that the new system $M^* = M + \Delta M$, $K^* = K + \Delta K$ has prescribed eigenvalues $\Lambda^* = \Lambda + \Delta\Lambda$ and eigenvectors $Y^* = Y + \Delta Y$.

It is seen from Eqs. (3) and (4) that the matrices M and K are functions of the matrices Λ and Y, e.g.,

$$M = M(\Lambda, Y) \tag{5}$$

$$K = K(\Lambda, Y) \tag{6}$$

We will assume that the elements of the matrices Λ and Y change and that these changes are small. We want to determine increments ΔM and ΔK of parameters from a new set of the eigenvalues Λ^* and the eigenvectors Y^*. We will use for that purpose the Taylor expansion.[1] According to this expansion, for sufficiently small increments of parameters, only the first term in the expansions need by retained, i.e.,

$$M^* = M + \mathfrak{D}_{\lambda T} M \cdot (\Delta\lambda \otimes I) + \mathfrak{D}_{yT} K \cdot (\Delta y \otimes I) \tag{7}$$

$$K^* = K + \mathfrak{D}_{\lambda T} M \cdot (\Delta\lambda \otimes I) + \mathfrak{D}_{yT} K \cdot (\Delta y \otimes I) \tag{8}$$